

Masses of Axial-Vector Resonances in a Linear Sigma Model with $N_f = 3$

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Abstract. We discuss an $N_f = 3$ linear sigma model with vector and axial-vector mesons (extended Linear Sigma Model - eLSM). We present first results regarding the masses of axial-vector mesons determined from the extended model.

Keywords: Chiral Lagrangian; sigma model; mesons; kaons.

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INTRODUCTION

The vacuum phenomenology of low-energy mesons can be described in a variety of approaches using the linear [1] and non-linear [2] realisations of the chiral symmetry. The linear realisation of the chiral symmetry (linear sigma model) has, for example, been used in Refs. [3, 4] in order to describe non-strange hadrons in the energy region up to approximately 1.7 GeV (see also Ref. [5] and Refs. therein). However, this energy region contains further experimentally well-established resonances [6], such as those composed solely of strange quarks, or those with an admixture of strange quark fields. These resonances are important for the description of meson phenomenology in vacuum and at non-zero temperatures and are expected to play an important role in the restoration of the chiral symmetry. Therefore, a more complete phenomenological description of mesons requires an extension of the model to $N_f = 3$. Additionally, the data regarding strange mesons is both precise and abundant thus offering more constraints for a phenomenological approach than the data regarding the non-strange mesons. In Ref. [7], we have outlined such an extension of the $N_f = 2$ linear sigma model with vector and axial-vector degrees of freedom, presented in Ref. [3], to $N_f = 3$ (extended Linear Sigma Model - eLSM). In this paper, we report on first results regarding axial-vector meson masses from eLSM.

The paper is organised as follows: in Sec. 2 we present the model Lagrangian and its implications and in Sec. 3 we summarise our results.

THE MODEL

We use an $N_f = 3$ linear sigma model with global chiral invariance in the following form [3, 7]:

$$\begin{aligned} \mathcal{L} = & \text{Tr}[(D^\mu \Phi)^\dagger (D^\mu \Phi)] - m_0^2 \text{Tr}(\Phi^\dagger \Phi) \\ & - \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \text{Tr}(\Phi^\dagger \Phi)^2 \\ & - \frac{1}{4} \text{Tr}[(L^{\mu\nu})^2 + (R^{\mu\nu})^2] + \frac{m_1^2}{2} \text{Tr}[(L^\mu)^2 + (R^\mu)^2] \\ & + \text{Tr}[H(\Phi + \Phi^\dagger)] + c(\det \Phi + \det \Phi^\dagger) \\ & - 2ig_2 (\text{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \text{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\ & - 2g_3 [\text{Tr}(\{\partial_\mu L_\nu + \partial_\nu L_\mu\} \{L^\mu, L^\nu\}) \\ & + \text{Tr}(\{\partial_\mu R_\nu + \partial_\nu R_\mu\} \{R^\mu, R^\nu\})] \\ & + \frac{h_1}{2} \text{Tr}(\Phi^\dagger \Phi) \text{Tr}[(L^\mu)^2 + (R^\mu)^2] \\ & + h_2 \text{Tr}[(\Phi R^\mu)^2 + (L^\mu \Phi)^2] + 2h_3 \text{Tr}(\Phi R_\mu \Phi^\dagger L^\mu), \end{aligned} \quad (1)$$

where

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{(\sigma_N + a_0^0) + i(\eta_N + \pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_S^+ + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0) + i(\eta_N - \pi^0)}{\sqrt{2}} & K_S^0 + iK^0 \\ K_S^- + iK^- & \bar{K}_S^0 + i\bar{K}^0 & \sigma_S + i\eta_S \end{pmatrix} \quad (2)$$

is a matrix containing the scalar and pseudoscalar degrees of freedom and

$$\begin{aligned} L^\mu &= \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} + \frac{f_{1N} + a_1^0}{\sqrt{2}} & \rho^+ + a_1^+ & K^{*+} + K_1^+ \\ \rho^- + a_1^- & \frac{\omega_N - \rho^0}{\sqrt{2}} + \frac{f_{1N} - a_1^0}{\sqrt{2}} & K^{*0} + K_1^0 \\ K^{*-} + K_1^- & \bar{K}^{*0} + \bar{K}_1^0 & \omega_S + f_{1S} \end{pmatrix}^\mu, \\ R^\mu &= \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} - \frac{f_{1N} + a_1^0}{\sqrt{2}} & \rho^+ - a_1^+ & K^{*+} - K_1^+ \\ \rho^- - a_1^- & \frac{\omega_N - \rho^0}{\sqrt{2}} - \frac{f_{1N} - a_1^0}{\sqrt{2}} & K^{*0} - K_1^0 \\ K^{*-} - K_1^- & \bar{K}^{*0} - \bar{K}_1^0 & \omega_S - f_{1S} \end{pmatrix}^\mu \end{aligned} \quad (3)$$

are, respectively, the left-handed and right-handed matrices containing the vector and axial-vector degrees of

freedom. Also, $D^\mu \Phi = \partial^\mu \Phi - ig_1(L^\mu \Phi - \Phi R^\mu)$ is the covariant derivative; $L^{\mu\nu} = \partial^\mu L^\nu - \partial^\nu L^\mu$ and $R^{\mu\nu} = \partial^\mu R^\nu - \partial^\nu R^\mu$ are, respectively, the left-handed and right-handed field strength tensors; the term $\text{Tr}[H(\Phi + \Phi^\dagger)]$ [$H = 1/2 \text{diag}(h_{0N}, h_{0N}, \sqrt{2}h_{0S})$, $h_{0N} = \text{const.}$, $h_{0S} = \text{const.}$] explicitly breaks chiral symmetry due to nonzero quark masses, and the term $c(\det \Phi + \det \Phi^\dagger)$ describes the $U(1)_A$ anomaly [8].

As in Ref. [3], in the non-strange sector, we assign the fields $\vec{\pi}$ and η_N to the pion and the $SU(2)$ counterpart of the η meson, $\eta_N \equiv (\bar{u}u + \bar{d}d)/\sqrt{2}$. The fields ω_N^μ and $\vec{\rho}^\mu$ represent the $\omega(782)$ and $\rho(770)$ vector mesons, respectively, and the fields f_{1N}^μ and \vec{a}_1^μ represent the $f_1(1285)$ and $a_1(1260)$ mesons, respectively. In the strange sector, we assign the K fields to the kaons; the η_S field is the strange contribution to the physical η and η' fields; the ω_S , f_{1S} , K^* and K_1 fields correspond to the $\phi(1020)$, $f_1(1420)$, $K^*(892)$, and $K_1(1270)$ mesons, respectively. In accordance with Ref. [3], where the scalar $\bar{q}q$ states were found in the energy region above 1 GeV, we assign the scalar kaon K_S to the physical $K_0^*(1430)$ state. The preliminary results from our extended model, Eq. (1), seem to point to the predominantly strange and non-strange sigma states to be above 1 GeV as well [7] (these states arise from the mixing of the pure quarkonium state σ_N and an the pure glueball state σ_S).

In order to implement spontaneous [3, 9] breaking of the chiral symmetry in vacuum by the quark condensate, we shift the σ_N and σ_S fields by their respective vacuum expectation values ϕ_N and ϕ_S .

The spontaneous symmetry breaking results in η_N - f_{1N} and $\vec{\pi}$ - \vec{a}_1 mixings: $-g_1\phi_N(f_{1N}^\mu\partial_\mu\eta_N + \vec{a}_1^\mu \cdot \partial_\mu\vec{\pi})$ [3] as well as in η_S - f_{1S} , K_S - K^* , and K - K_1 mixings: $-\sqrt{2}g_1\phi_S f_{1S}^\mu\partial_\mu\eta_S$, $ig_1(\sqrt{2}\phi_S - \phi_N)(\bar{K}^{*\mu 0}\partial_\mu K_S^0 + K^{*\mu -}\partial_\mu K_S^+)/2 + ig_1(\phi_N - \sqrt{2}\phi_S)(K^{*\mu 0}\partial_\mu \bar{K}_S^0 + K^{*\mu +}\partial_\mu K_S^-)/2$ and $-g_1(\phi_N + \sqrt{2}\phi_S)(K_1^{\mu 0}\partial_\mu \bar{K}^0 + K_1^{\mu +}\partial_\mu K^-)/2 + \text{h.c.}$, respectively. Note that our Lagrangian is real despite the imaginary K_S - K^* coupling because the K_S - K^* mixing term is equal to its hermitian conjugate and therefore real.

The mixing terms are removed similarly to the way described in Ref. [3], with suitable shifts of the vector field K^* and the axial-vector fields concerned: $f_{1N,S}^\mu \rightarrow f_{1N,S}^\mu + w_{f_{1N,S}}\partial^\mu\eta_{N,S}$; $\vec{a}_1^\mu \rightarrow \vec{a}_1^\mu + w_{a_1}\partial^\mu\vec{\pi}$; $K^{*\mu 0} \rightarrow K^{*\mu 0} + w_{K^*}\partial^\mu K_S^0$; $K^{*\mu +} \rightarrow K^{*\mu +} + w_{K^*}\partial^\mu K_S^+$; $\bar{K}^{*\mu 0} \rightarrow \bar{K}^{*\mu 0} + w_{K^*}^*\partial^\mu \bar{K}_S^0$; $K^{*\mu -} \rightarrow K^{*\mu -} + w_{K^*}^*\partial^\mu K_S^-$; $K_1^{\mu 0} \rightarrow K_1^{\mu 0} + w_{K_1}\partial^\mu K^0$ (and h.c. for K_1), where the w constants are defined in such a way that the mentioned mixing terms are removed from the Lagrangian: $w_{f_{1N}} = w_{a_1} = g_1\phi_N/m_{a_1}^2$ (one obtains $w_{f_{1N}} = w_{a_1}$, as in Ref. [3]), $w_{f_{1S}} = \sqrt{2}g_1\phi_S/m_{f_{1S}}^2$, $w_{K^*} = ig_1(\phi_N - \sqrt{2}\phi_S)/(2m_{K^*}^2)$

and $w_{K_1} = g_1(\phi_N + \sqrt{2}\phi_S)/(2m_{K_1}^2)$.

Subsequently, as in Ref. [3], the fields $\eta_{N,S}$, $\vec{\pi}$, K_S and K are no longer canonically normalised. In order to obtain canonical normalisation, we introduce renormalisation constants (coefficients) of these wave functions labelled $Z_{\eta_{N,S}}$ for $\eta_{N,S}$, Z_π for $\vec{\pi}$, Z_{K_S} for K_S and Z_K for K (note that these coefficients do not contain loop corrections and can thus have a value larger than one, see Table 1 for the values of Z_π). We obtain the following formulas:

$$Z_\pi \equiv Z_{\eta_N} = \frac{m_{a_1}}{\sqrt{m_{a_1}^2 - g_1^2\phi_N^2}} \quad (4)$$

$$Z_{\eta_S} = \frac{m_{f_{1S}}}{\sqrt{m_{f_{1S}}^2 - 2g_1^2\phi_S^2}} \quad (5)$$

$$Z_K = \frac{2m_{K_1}}{\sqrt{4m_{K_1}^2 - g_1^2(\phi_N + \sqrt{2}\phi_S)^2}} \quad (6)$$

$$Z_{K_S} = \frac{2m_{K^*}}{\sqrt{4m_{K^*}^2 - g_1^2(\phi_N - \sqrt{2}\phi_S)^2}}. \quad (7)$$

For the non-strange and strange condensates, we then have $\phi_N = Z_\pi f_\pi$ [3] and analogously $\phi_S = Z_K f_K/\sqrt{2}$, where $f_\pi = 92.4$ MeV and $f_K = 155$ MeV $/\sqrt{2}$ are, respectively, the pion and kaon decay constants.

Additionally to Eq. (6), we obtain two more formulas for Z_K from $m_{f_{1S}}^2 - m_{\omega_S}^2$ and $m_{K_1}^2 - m_{K^*}^2$:

$$Z_K = \frac{1}{f_K} \sqrt{\frac{m_{f_{1S}}^2 - m_{\omega_S}^2}{g_1^2(Z_\pi) - h_3(Z_\pi)}} \quad (8)$$

$$Z_K = \frac{m_{K_1}^2 - m_{K^*}^2}{Z_\pi f_\pi f_K [g_1^2(Z_\pi) - h_3(Z_\pi)]}. \quad (9)$$

Therefore, in order to be consistent, the values of Z_K have to simultaneously fulfill three equations: (6), which is the definition of Z_K , (8) and (9). We note that Eqs. (6), (8) and (9), in addition to m_{ω_S} , $m_{f_{1S}}$, m_{K^*} and m_{K_1} , also contain m_ρ and m_{a_1} present in parameters g_1 and h_3 (see Ref. [3]):

$$g_1(Z_\pi) = \frac{m_{a_1}}{Z_\pi f_\pi} \sqrt{1 - \frac{1}{Z_\pi^2}} \quad (10)$$

$$h_3(Z_\pi) = \frac{m_{a_1}^2}{Z_\pi^2 f_\pi^2} \left(\frac{m_\rho^2}{m_{a_1}^2} - \frac{1}{Z_\pi^2} \right). \quad (11)$$

Hence, we need to determine masses that are to be implemented in the Eqs. (6), (8) and (9), i.e., the masses that should correspond to the experimental data (up to loop corrections to our tree-level masses, with corrections not expected to be large in the case of our resonances). The

ρ and K^* states are well-established quarkonia [10]; the states currently present in our model are $\bar{q}q$ states [3] and thus we set the ρ and K^* masses to the PDG values: $m_\rho = 775.49$ MeV and $m_{K^*} = 891.66$ MeV. We assign our $\omega_S \equiv \bar{s}s$ state to the physical $\phi(1020)$ resonance because this resonance is known to be predominantly an $\bar{s}s$ field, although with a small admixture of the non-strange quarks. Our Lagrangian does not implement $\bar{s}s - \bar{n}n$ mixing in the isosinglet vector channel and thus, as a first approximation, we set the ω_S mass to the PDG value: $m_{\omega_S} = 1019.455$ MeV.

Given the assignment of the states in our model to the physical states, one would usually also set $m_{f_{1S}} = 1426.4 \pm 0.9$ MeV, $m_{K_1} = 1272 \pm 7$ MeV and $m_{a_1} = 1230 \pm 40$ MeV [6]. The latter value is merely an "educated guess" [6]. Therefore, in order to simultaneously fulfill Eqs. (6), (8) and (9), we can relax the interval for m_{a_1} and search for a suitable value in the region 1.1 - 1.3 GeV while retaining the values of m_{K_1} and $m_{f_{1S}}$ in the vicinity of the PDG data. In this way, the hypothesis of $K_1(1270)$ and $f_1(1420)$ as predominantly $\bar{q}q$ states is tested; obtaining their masses in the vicinity of the experimental data would be an indication that this hypothesis is justified. Note also that it is actually not possible to simultaneously fulfill Eqs. (6), (8) and (9) if $m_{f_{1S}}$, m_{K_1} and m_{a_1} are set to their exact respective values quoted by the PDG.

We broaden the interval $Z_\pi = 1.67 \pm 0.2$ found in Ref. [3] to $1.1 \leq Z_\pi \leq 1.9$ in order to obtain the axial-vector masses for more general parameter values. Subsequently, by enforcing the equality of the three Eqs. (6), (8) and (9), obtain constraints on m_{K_1} , $m_{f_{1S}}$ and m_{a_1} . We vary m_{a_1} between 1.1 and 1.3 GeV and look for m_{K_1} and $m_{f_{1S}}$ that are as close as possible to the PDG data [$m_{K_1}^{\text{PDG}} = (1272 \pm 7)$ MeV and $m_{f_{1S}}^{\text{PDG}} = (1426.4 \pm 0.9)$ MeV]. We find the results presented in Table 1.

TABLE 1. Values of m_{a_1} , $m_{f_{1S}}$ and m_{K_1} that, for a given Z_π , lead to the same value of Z_K from Eqs. (6), (8) and (9).

Z_π	m_{a_1} (MeV)	m_{K_1} (MeV)	$m_{f_{1S}}$ (MeV)
1.1	1142	1276	1423
1.2	1144	1276	1421
1.3	1147	1277	1420
1.4	1152	1279	1419
1.5	1157	1281	1418
1.6	1163	1283	1416
1.7	1170	1285	1413
1.8	1180	1289	1411
1.9	1191	1292	1406

Thus, m_{a_1} is about 50-100 MeV smaller than the PDG value [6], as was also obtained in the $N_f = 2$ Lagrangian

in Ref. [3]. However, we obtain values of m_{K_1} and $m_{f_{1S}}$ that are very close to the experimental data, with both mass values deviating from the PDG data by approximately 20 MeV at the most hence favouring the hypothesis that these states are predominantly of $\bar{q}q$ nature.

CONCLUSIONS

We have presented a linear sigma model with vector and axial-vector degrees of freedom that, in our approach, has been extended to $N_f = 3$. Implementing the spontaneous symmetry breaking in the model yields not only the known η_N - f_{1N} and $\bar{\pi}$ - \bar{a}_1 mixings [3] but also the η_S - f_{1S} , K_S - K^* and K - K_1 mixings. Removing the non-diagonal terms in the Lagrangian and subsequently bringing the $\eta_{N,S}$, $\bar{\pi}$, K_S and K derivatives to the canonical form leads us to, among others, define the kaon renormalisation coefficient Z_K . Besides its definition formula, Eq. (6), Z_K also possesses two other formulas obtained from the difference of the strange axial-vector and vector mass terms $m_{f_{1S}}^2 - m_{\omega_S}^2$ and $m_{K_1}^2 - m_{K^*}^2$, Eqs. (8) and (9). Setting m_ρ , m_{K^*} and m_{ω_S} to their PDG values and enforcing the equality of the three mentioned Z_K formulas yields constraints on m_{a_1} , m_{K_1} and $m_{f_{1S}}$. We leave m_{a_1} free due to its large decay width. Consequently, we obtain $1276 \text{ MeV} \leq m_{K_1} \leq 1292 \text{ MeV}$ and $1406 \text{ MeV} \leq m_{f_{1S}} \leq 1423 \text{ MeV}$ which is in a very good agreement with the experimental data [6]. Note that this result favours the $K_1(1270)$ and $f_1(1420)$ resonances as predominantly $\bar{q}q$ states.

Calculations of all meson masses and, subsequently, of the decay widths of the resonances in the Lagrangian (1) present an outlook of our approach [11].

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